DOCUMENT RESUME

ED 139 092 EA 009 505

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TITLE	Further Studies on the Evaluation of Internal Efficiency of Educational Systems: A Symposium. Current Surveys and Research in Statistics.
INSTITUTION	United Nations Educational, Scientific, and Cultural Organization, Paris (France). Div. of Statistics on Education.
REPORT NO	CSR-E-3
PUB DATE	72
NOTE	62p.; For a related document, see EA 009 503 ; Some parts may be marginally legible due to print quality
EDRS PRICE	MF-$0.83 HC-$3.50 Plus Postage.
DESCRIPTORS	Bibliographies; *Cost Effectiveness; *Educational Finance; Efficiency; Elementary Secondary Education; *Mathematical Models; Prediction; *Research Methodology; *Statistical Analysis; Tables (Data)

ABSTRACT

This publication describes some of the work of the UNESCO Division of Statistics on Education on the evaluation of the internal efficiency of educational systems that followed from the 1970 International Conference on Education. The first section contains a brief summary of the conclusions of the "Expert Meeting on the Evaluation of Internal Efficiency of Educational Systems" held by UNESCO in January 1972. The second section contains a paper by Z. Hellwig entitled "Wastage Prediction Method," which was prepared for discussion at the Expert Meeting. The third section contains some technical notes by D. Blot on Professor Hellwig's paper. The fourth section contains a paper by A. Brimer entitled "The Quantification of School Events Related to Educational Wastage," which was also prepared for discussion at the Expert Meeting. (JG)

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ED139092

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Further studies on the evaluation of internal efficiency of educational systems: A symposium

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P R E F A C E

This study is the fourth in a new series of reports on the current work in progress in the various Divisions of the Office of Statistics of Unesco. The major work of the Office of Statistics is presented formally in the Unesco Statistical Yearbooks, the Statistical Reports and Studies series, the reference documents for Ministerial Conferences and in other official publications of Unesco and the United Nations. The purpose of this new series is to disseminate, in an informal manner, the results of the current research studies and surveys being undertaken by the Office, without the delays inherent in more formal publications.

It is intended that this series, which will appear in both English and French, will be as wide ranging as is the work of the Office. It will include statistical reviews of selected topics, analytical enquiries, examinations of methodological problems, research studies, etc. It reflects a new and exciting stage in the development of the work of the Office, and it is to be hoped that the series will be of considerable interest to persons working in the Unesco subject field as well as to the practising statisticians. Any comments about the studies would be welcomed and should be sent to the Director, Office of Statistics, UNESCO.

This present study gives an account of the recent work in the Division of Statistics on Education on the evaluation of internal efficiency of educational systems, following the work on the statistical measurement of wastage carried out by the Office of Statistics for the 1970 International Conference on Education. One subsequent paper "Costing Educational Wastage - a Pilot Simulated Study" by D. Berstecher was issued as CSR-E-1 in the same series. Two further papers by Z. Hellwig and M.A. Brimer are contained in this follow-up study as well as some technical notes by D. Blot on Professor Hellwig's paper. All these papers were discussed at an Expert Meeting held in Unesco and the present study includes the report of this meeting with its implications for the future work programme of the Office together with a statistical bibliography.

Expert Meeting on the Evaluation of Internal Efficiency of Educational Systems

(Paris, 17 - 21 January, 1972)

SUMMARY OF CONCLUSIONS

An expert meeting on the Evaluation of Internal Efficiency of Educational Systems was organized by the Division of Statistics on Education, Office of Statistics, at Unesco Headquarters, from 17 to 21 January 1972. The purpose of the meeting was to consider the next steps to be taken for the implementation of the statistical aspects of the Recommendation N° 66 adopted by the International Conference on Education (32nd Session, July 1970), concerning the improved effectiveness of educational systems particularly through reduction of wastage at all levels of education. A selected group of specialists engaged in work in this field was invited to discuss the work undertaken or planned in the following specific topics :-

(a) How to achieve greater accuracy in the collection of data;

(b) How to check the reliability of wastage indices and the deductions to be made from them;

(c) The elaboration of techniques for the assessment of wastage in school systems without repetition or drop-out;

(d) The elaboration of indicators of wastage for the purpose of simulation on the basis of alternative hypotheses;

(d) The nature and incidence of wastage in higher education.

Three consultants presented their papers :-

1. "Costing Educational Wastage", Dr. D. Berstecher, Institut für Bildungsforschung in der Max-Planck-Gesellschaft, Berlin.

2. "Wastage Prediction Method", Professor Z. Hellwig, Professor of Econometrics and Mathematical Statistics, Graduate School of Economics, Wroclaw, Poland.

3. "The Quantification of School Events related to Educational Wastage", Professor M.A. Brimer, University of Bristol, United Kingdom.

Two further outside specialists also participated at the meeting : Mr. D. Blot, who represented O.E.C.D. and Professor M. Debeauvais, Department of Education, University of Paris, VIII. Mr. Nguyen Tinh took part as an observer from the Institut d'étude du développement économique et social and Mr. P. Bernard, currently working as a consultant on school retardation, participated as an observer.

In addition to the members of the Division of Statistics on Education, the sessions were attended by members of the IIEP, EP, EDV, EDS and SS.

Besides the three papers presented by the consultants above-mentioned, the participants at the meeting had before them a Secretariat Background Paper summarizing the main lines of the Statistical Office's current work and proposed work programme in this area and the result of preliminary tests of some of the working assumptions used in the evaluation of internal efficiency. The application of the Wastage Prediction Methods presented to data for three countries with different functioning patterns was also summarized in this paper with a view to exploring the goodness of fit of reconstructed cohorts under changing conditions as a verification of the plausibility of the indices and conclusions derived.

In addition, three earlier background papers were referred to :

1. "The Statistical Measurement of Educational Wastage", UNESCO.

2. "A Modern System of Educational Statistics : The Matrix Method", by J. de Bruyn.

3. "A School Cohort Coding System", by L. Goldstone.

A. Main Conclusions

The meeting focussed on two major issues :

I. The improvement of indicators and analyses to be derived from the type of data currently available.

II. The need to obtain a more detailed and comprehensive efficiency measure.

As regards item I, the following conclusions were established :-

1. The basic plausibility of the assumptions being adopted for the construction of cohorts on the basis of observed rates was admitted.

2. The considerably better insight into the internal functioning of educational systems provided through flow analysis as opposed to "apparent methods" was recognized and the misleading conclusions led to by the latter emphasized.

3. The need for supplementary indices was expressed as well as for the devising of typologies of wastage profiles. The need for investigation of the causal aspects, particularly social backgrounds of unsuccessful pupils, teachers' qualifications and . In this connection, the convenience of studies aiming at comparing different zones, districts or regions, inside a country was stressed, taking into account the existing constraints and limitations. These studies should lead to a typology of countries according to their efficiency features.

4. The interest provided by the simulation work on costing wastage was underlined and the desirability of application to more refined situations recommended. The cost of measures to improve efficiency was deemed deserving of careful investigation.

5. The participants expressed their appreciation of methods for predic as verification of the correct reconstruction of school cohorts and as forecasting tools. Several improvements were, however, suggested, especially the consideration of the rate of increase of the number of newcomers entering the system in relevant cases.

In connection with item II, the improvement of detail and coverage for a correct evaluation of efficiency, the main points discussed were:-

1. The need for a closer identification of wastage events was underlined. In this connection, the evaluation of effectiveness of the educational system at the different stages and levels was judged to be of paramount importance. The establishment of enrolment flows through the various cycles, including dropouts within the school year, appeared highly desirable.

2. The consideration of certain parameters, so far not available (such as mortality, transfers within the system, temporary interruption and later resumption, etc.) were recommended.

3. The identification of transition patterns between the various cycles and the efficiency features in higher education were judged to call for special sampling studies in selected countries.

4. The analysis of school careers.

B. Follow-up

The participants agreed upon the following stpes to be taken :-

1. Methodological studies should be undertaken with a view to reaching an acceptable precision of diagnosis in those countries where the minimum required data is not available at present. Thus margins of error between actual flows, reconstructed cohorts and apparent methods should be estimated by using enrolment by grade only and also by using age data when available.

2. Case studies should be undertaken specially oriented towards the testing of some simple cohort coding system.

3. The above studies should permit the checking of indices derived by cruder methods and the estimation of their level of accuracy. Factors such as availability of places and successful completion should be taken into account in these exercises.

4. Following the publication of the paper by D. Berstecher "Costing Educational Wastage" in the new Office of Statistics series Current Surveys and Research in Statistics (CSR-E-1), it was agreed that a revised version of the papers by Professors Hellwig and Brimer, including a summary of Mr. Blot's comments on Professor Hellwig's paper, and Notes from the Secretariat, together with a selected annotated statistical bibliography, on the subject, should be published in the same series as "Further Studies in the Evaluation of Internal Efficiency of Educational Systems" CSR-E-3.

5. The next session of the International Conference on Education should be informed of the results of the Expert Meeting organized by the Office of Statistics.

WASTAGE PREDICTION METHOD

Z. Hellwig

1. INTRODUCTION

One of the most important factors in the technical, economic, social and cultural development of a contemporary state is, beyond doubt, the skill, experience and knowledge of its people. This fact is widely acknowledged and therefore all governments and many international institutions do their best to stimulate all forms of educational activity. At the same time, however, a lot of criticism has recently been raised against a traditional and obsolete organization of educational systems. The most fundamental and well-founded criticism has been aimed at the lengthy and still expanding duration of educational processes, accompanied by the high cost of training and the low efficiency of an educational system. To meet this criticism, serious efforts have been undertaken to investigate the main causes of low performance in modern schooling.

2. PROBLEM OF EDUCATIONAL WASTAGE

In 1970 a statistical study "The Statistical Measurement of Educational Wastage" was issued by the UNESCO Office of Statistics. In this study a very interesting idea was put forward : that the quickest and cheapest way towards improvement of the efficiency of an educational system lay in a reduction of wastage, which in its turn is composed of repetition and drop-out. In Chapter I, page 5 of the study we find the following important sentence : "Planners are aware of the fact that intake capacity and successful completion in any educational system are directly related to repetition and holding power. The inter-relationship of these factors and their combined sequence determine the dynamics of most systems and play a key role in true unit cost of each successful completer, whether this is expressed in monetary or non-monetary terms". The document from which this excerpt is quoted also contains all the necessary definitions and some bibliographic references.

In the same publication, a methodological proposal has been submitted according to which successes (promotions) and failures (repetitions and drop-puts) could be treated like demographic acts of birth and death. It is well known that demography has developed valuable and efficient methods of analysing different aspects of demographic movements. The same methods are also valid when applied in the field of educational wastage. Hence, different components of educational wastage behave according to the same statistical laws which govern all discrete stochastical processes belonging to the class of so-called Markov processes. In the study quoted above many international comparisons were made, based on original verified statistical data, collected with the specific purpose of highlighting features of educational wastage.

The importance of the wastage problem in the pursuit of increasing efficiency of schooling has won international recognition. This general support helped immensely in expanding permanent statistical studies of the wastage problem which received a new impetus after the 32nd session of the International Conference on Education in 1970.

3. ASSUMPTIONS

The wastage problem was examined under certain assumptions and premises. We will enumerate here only the most important of these :

1° The aptitude of pupils for subjects taught in school is normally distributed. The distributions of pupils can vary over subjects with respect to parameters but not to the shape of distributions.

2° There are three possible outcomes of the teaching process: (A) passing to the next grade, (B) repetition of a given grade, and (C) drop-out. Let us agree that A, B, C are random events. Hence we may reasonably accept that $a = P(A)$, $b = P(B)$, $c = P(C)$ are appropriate probabilities of these events.

3° For the single promotion act we have $A \vee B \vee C = U$ where U stands for a certain event. Hence

$$P(A) + P(B) + P(C) = 1$$

4° Probabilities a, b, c depend on numbers i, j, where i stands for the number of the current i-th year and j denotes the number of the j-th grade.

emphasize this fact the following notation will be used :

$$a_{ij},\ b_{ij},\ c_{ij};\quad i = 1,2,\ldots m,\quad j = 1,2,\ldots n$$

where m denotes the maximal duration of the schooling period in the worst case [1] and n means the number of grades. Hence, taking into account 3° one gets :

$$a_{ij} + b_{ij} + c_{ij} = 1.$$

It follows from this equality that there is no possibility for a student, no matter how apt or clever he is, to pass in one academic year from the grade j to the grade $j + l$ if $l = 2,3,\ldots n-j$.

5° There is no intragrade external enrolment.

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[1] *Or, if we express this in a slightly more formal way : m fulfills the following inequality :*

$b \quad b_{2j} \ldots b_{mj} < \varepsilon$ *where* ε *is a positive number close to zero (see also [2]).*

6° Students who have already repeated a certain grade m times and have to repeat this grade once more must leave the school. Hence m + 1 - th repetition is according to law equivalent to an act of dropping out.

7° Let

$$P_{ij}\ (B/B)$$

be a probability of repeating j-th grade in i+1st year given in i-th year the same grade has also been repeated.

Similarly, let $P_{ij}\ (A/B)$ be a probability of promotion in the year i+1-st from the grade j-th to the grade j+1-st given in the year i-th the grade j-th has been repeated.

We assume that

$$P_{ij}\ (A/B) = a_{ij}$$

and

$$P_{ij}\ (B/B) = b_{ij}$$

This assumption means that random events A and B are stochastically independent. The same will be asserted with regard to the event C. In other words, all three events A, B, C will be looked upon as stochastically independent (see /4/).

Also let

$$P_{ij}(B/B_2),\ P_{ij}(B/B_3),\ldots,\ P_{ij}(B/B_{i-1})$$

be probabilities of repeating j-th grade given the same grade has been already repeated once, twice, ..., or i-1 times respectively. In exactly the same way we introduce the following notation:

$$P_{ij}(A/B_2),\ P_{ij}(A/B_3),\ldots,\ P_{ij}(A/B_{i-1})$$

The following assumption is being made

$$P_{ij}(A/B_1) = P(A/B_2) = \ldots = P_{ij}(A/B_{i-1}) = a_{ij}$$

$$P_{ij}(B/B_1) = P(B/B_2) = \ldots = P_{ij}(B/B_{i-1}) = b_{ij}$$

8° Since the human population changes very slowly in time we are justified in assuming that

$$a_{ij} = a_j,$$

$$b_{ij} = b_j,$$

$$c_{ij} = c_j.$$

4. DATA

Suppose in a given year i-1 a group of pupils entered an educational system. Let it be called a cohort. If we wish to learn of all the problems of pupils passing from one grade to another we would have to watch the cohort through all its life, i.e. from the very first moment of enrolment of the group of pupils till the last member of the cohort leaves the schooling system. Watching a cohort is too cumbersome and too costly for practical purposes for most countries. Besides, official educational statistics rarely report on the life of a cohort but simply provide us with data concerning the number of pupils in a given year dispersed over grades.

A commonly used statistical proforma is as follows :

SCHOOL YEAR		GRADES					
		1	2	...	j	...	n
1	Total enrolment						
	of which repeaters						
2	Total enrolment						
	of which repeaters						
.	Total enrolment						
:	of which repeaters						
i	Total enrolment				D_{ij}		
	of which repeaters				B_{ij}		
.	Total enrolment						
:	of which repeaters						

FIGURE 1 (see /7/)

5 SYMBOLS[1]

On the basis of the proforma the following variables are defined:

A_{ij} - number of pupils promoted in the i-th year from the grade j-th to the grade j+1-st.

B_{ij} - number of pupils repeating in the i-th year the j-th grade.

C_{ij} - number of pupils dropping out in the i-th year from the j-th grade.

D_{ij} - number of pupils in the i-th year in the j-th grade.

Here are some basic relations connecting the variables concerned:

$$D_{ij} = A_{ij} + B_{i+1,j} + C_{ij} \qquad (1)$$

$$D_{ij} = A_{i-1,j-1} + B_{ij} \qquad (2)$$

These variables can also be transformed into the following set of coefficients, which play a key role in educational wastage analysis :

$$\hat{a}_{ij} = \frac{A_{ij}}{D_{ij}} \qquad \text{(promotion rate)} \qquad (3)$$

$$\hat{b}_{ij} = \frac{B_{i+1,j}}{D_{ij}} \qquad \text{(repetition rate)} \qquad (4)$$

$$\hat{c}_{ij} = \frac{C_{ij}}{D_{ij}} \qquad \text{(drop-out rate)} \qquad (5)$$

The following equality results immediately from (1) - (5) :

$$\hat{c}_{ij} = 1 - (\hat{a}_{ij} + \hat{b}_{ij}).$$

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1 *In many papers (e.g. /1/, /7/) some other symbols have been used : E for enrolment, P for promotion and R for repetition. Since, however, E is also widely applied for expected value, P for probability and R (or r) for correlation coefficient, therefore to avoid possible confusion the "neutral" symbols have been adopted here.*

Coefficients $\hat{a}_{ij}$, $\hat{b}_{ij}$, $\hat{c}_{ij}$ are estimates of respective probabilities a_{ij}, b_{ij}, c_{ij}. These estimates possess all the important properties required from any well-defined statistical parameters : they are consistent, unbiased, sufficient and efficient. They are normally distributed and hence for a given number α , positive and close to unity, and one can easily determine a confidence interval.

Let us take, for example, the promotion rate $\hat{a}_{ij}$.

We obtain

$$P\{\hat{a}_{ij} - ts(\hat{a}_{ij}) < a_{ij} < \hat{a}_{ij} + ts(\hat{a}_{ij})\} = \alpha ,$$

where t depends on α and

$$s(\hat{a}_{ij}) = \sqrt{\frac{\hat{a}_{ij}(\hat{b}_{ij} + \hat{c}_{ij})}{D_{ij}}}$$

6. STABILITY

According to the assumption 7° in § 3 probabilities a_{ij}, b_{ij}, c_{ij} do not change in time. In other words, $a_{ij} = a_j$, (and similarly $b_{ij} = b_j$, $c_{ij} = c_j$).

This assumption can easily be verified on the basis of empirical statistical data. In table 2 (see Appendix) statistical data concerning estimates $\hat{a}_{ij}$, $\hat{b}_{ij}$, $\hat{c}_{ij}$ are presented. They describe the functioning of an educational system in Colombia. The reader will notice at once that all the parameters involved exhibit only small variations. This regularity remains valid with respect to all countries with developed and well-functioning educational systems. Hence the assumption 8° about the stability of promotion, repetition and drop-out ratios may be accepted as being duly verified.

7. INDEPENDENCE

The second crucial assumption which deserves special attention states that the changes of a pupil's success or failure in a given year do not depend upon his performance in the previous year. This is the content of the assumption 7° in § 3 which states that

$$P_{ij}(A/B) = a_{ij} = a_j$$

$$P_{ij}(B/B) = b_{ij} = b_j$$

The verification of this assumption cannot be carried out on the basis of data provided by statistical reports when they are presented in the proforma as has been given in figure 1. The most convenient and appropriate method which should be recommended for the purpose of checking independence hypotheses is connected with the application of a so-called coefficient of dependence (see /6/). For the sake of a pure demonstration of

how this measure of independence works, let us examine the following example. Figure 2 shows a twofold classification table.

Y \ X	A	B	$\sum$
I	620	150	770
II	165	65	230
$\sum$	785	215	1000

Figure 2

There are two categories of pupils : I and II. To the first category belong those pupils who are not repeating the grade. They are non-repeaters. To the II category belong the repeaters. In the next year of schooling some pupils will pass to the next grade and some will fail. All those who succeed in promoting to the higher grade are labelled as belonging to the group A, whereas all those remaining enter the group B. Let the number of pupils, located in the i-th row and in the j-th column, be denoted by the symbol n_{ij}. Hence

$$\sum_{i=1}^{2} n_{ij} = n_{j}$$

$$\sum_{j=1}^{2} n_{ij} = n_{i}$$

$$\sum_{i=1}^{2} \sum_{j=1}^{2} n_{ij} = \sum_{i=1}^{2} n_{i} = \sum_{i=1}^{2} n_{j} = n.$$

Let us introduce some additional symbols :

$$f_{ij} = \frac{n_{ij}}{n}$$

$$\underline{f}_{i} = \sum_{j=1}^{2} f_{ij}$$

$$\underline{\underline{f}}_{j} = \sum_{i=1}^{2} f_{ij}.$$

Hence

$$\sum_{i=1}^{2} \sum_{j=1}^{2} f_{ij} \quad \sum_{i=1}^{2} \underline{f}_{j} \quad \sum_{j=1}^{2} \underline{\underline{f}}_{j} = 1$$

We are ready now to set up the following table (Figure 3).

X / Y	A	B	$\sum$
I	f_{11} $\underline{\underline{f}}_1$ 1 $\underline{f}_1$	f_{12} $\underline{\underline{f}}_1$ $\underline{f}_2$	$\underline{\underline{f}}_1$ $\underline{f}_1$
II	f_{21} $\underline{\underline{f}}_2$ $\underline{f}_1$	f_{22} $\underline{\underline{f}}_2$ $\underline{f}_2$	$\underline{\underline{f}}_2$ $\underline{f}_2$
$\sum$	$\underline{f}_1$ $\underline{f}_1$	$\underline{f}_2$ $\underline{f}_2$	1 1

Figure 3

Here is a table (see Figure 4) where all the symbols have been substituted by specific numbers. These numbers have been calculated according to the formula presented above on the basis of data provided by the table shown in Figure 2. So we have :

$$f_{11} = \frac{620}{1000} = .620; \qquad f_{12} = \frac{150}{1000} = .150$$

$$f_{21} = \frac{165}{1000} = .165; \qquad f_{22} = \frac{65}{1000} = .065$$

$$f_{\underline{1}} = \frac{785}{1000} = .785; \qquad f_{\underline{2}} = \frac{215}{1000} = .215$$

$$f_{\overline{1}} = \frac{770}{1000} = .770; \qquad f_{\overline{2}} = \frac{230}{1000} = .230$$

Hence

X / Y	A	B	$\sum$
I	.620 / .604	.150 / .166	.770 / .770
II	.165 / .181	.065 / .049	.230 / .230
$\sum$	.785 / .785	.215 / .215	1.00 / 1.00

Figure 4.

Let us now introduce the following measure of stochastical dependence :

$$d = 2 \sum_{(i,j)\ M} (f_{ij} - f_{\underline{i}} \cdot f_{\overline{j}})$$

where M denotes a set of such pairs of indices (i,j) that $f_{ij} \neq f_{\underline{i}} f_{\overline{j}}$ (in the table shown in Figure 4 these cells, which correspond to elements of the set M, are marked with thick lines).

It can be proved (see /6/) that

$$0 \leq d \leq 1.$$

If d equals zero then the random variables X and Y are stochastically independent; if d equals unity then the random variables are closely related and one may expect that they are linked together by a relationship of a functional type.

In the case of the example considered we get

$$d = 2\,(.620 + .065 - .604 - .049) = .064$$

Since d is close to zero then the random variables X and Y can be considered as stochastically independent. There are, of course, other methods of checking if the random variables X and Y are independent. One of them, simple, convenient and well-known, is χ^2 - method, developed by R.A. Fisher.

Let us come back to our example. We say that random variables X, Y are independent when the following inequality holds :

$$\chi^2 \leq \chi_o^2$$

where

$$\chi^2 = n\left(\sum_{i=1}^{2}\sum_{j=1}^{2}\frac{n_{ij}^2}{n_{i\cdot}\,n_{\cdot j}} - 1\right)$$

and χ_o^2 is a positive number for which

$$P\,(\chi^2 > \chi_o^2) < \alpha$$

where α is a positive number close to zero.

To simplify the computational procedure we can introduce the following table (Figure 5) containing numbers

$$\frac{n_{ij}^2}{n_{i\cdot}\,n_{\cdot j}} \qquad i,j = 1,2.$$

i \ j	1	2	$\sum$
1	.636	.136	.772
2	.146	.085	.231
$\sum$	.782	.221	1.003

Figure 5.

For α = .05 and one degree of freedom it follows from χ^2 tables that

$$\chi^2_0 = 3{,}841.$$

Since

$$\chi^2 = n (1 . 003 - 1) = 1000 \times .003 = 3,$$

Therefore

$$\chi^2 < \chi^2_0$$

and we conclude, as before, that the random variables X and Y are stochastically independent.

As presented above, the verification of independence hypothesis does not create any statistical or numerical difficulties. There is, of course, one fundamental condition which always has to be fulfilled, namely that one should be provided with all the necessary statistical data. Since these data are not obtained through official reporting, special statistical investigation must be undertaken. It should be strongly emphasized that the necessary information could easily be obtained by means of random sampling. The most convenient and theoretically commendable technique consists in collecting statistical data through selecting a stratified sample. One can expect that a twofold stratification would provide satisfactory results and a size of sample not larger than one thousand items would certainly suffice. As long as statistical data for checking the independence hypothesis are not available one has to accept the fact that all consideration based on the assumption 7° in § 3 may turn out to be incorrect.

8. STANDARD MODEL OF EDUCATIONAL WASTAGE

In the paper [7] a method was presented which allows us to simulate the functioning of the educational system and exploit coefficient a_i, b_i and c_i to obtain a so-called flow-

diagram. Here is an example of suchaa diagram constructed on the basis of data concerning Colombia (Figures 6 and 8).

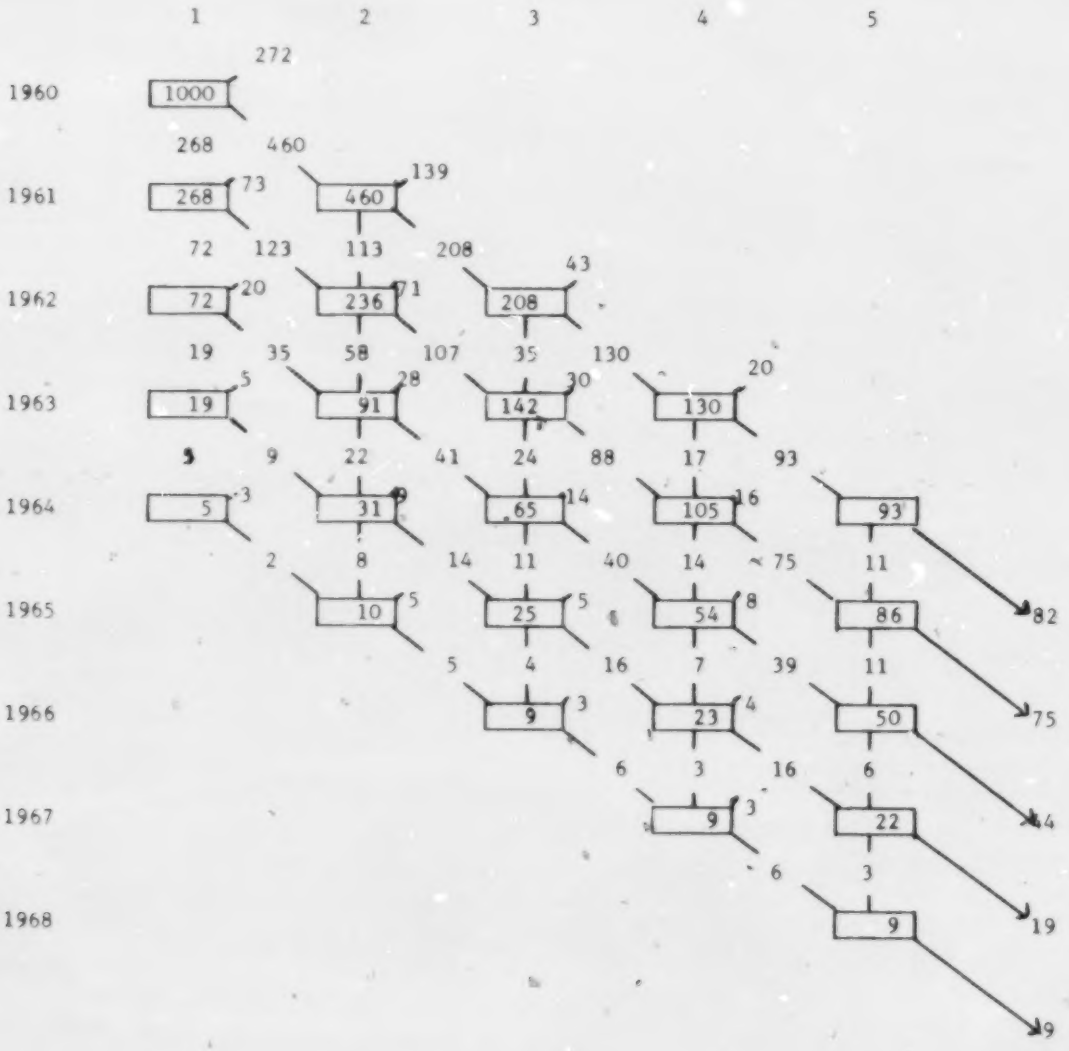

Figure 6.

The reader should notice at once that the flow-chart presented in figure 6 has been constructed on the basis of the assumptions 7° and 8° in the § 3. For the sake of simulation, all ratios were kept constant. Here is a table which contains all the parameters involved.

Parameters \ j	1	2	3	4	5
a_j	.460	.453	.622	.711	.876
b_j	.268	.245	.170	.133	.124
c_j	.272	.302	.208	.156	-

Figure 7.

9. RATE OF SCHOOLING EFFICIENCY

Let us sum up all the repetitions and drop-outs in the first column of the flow-diagram. We get

Repetitions	Drop-outs
268	272
72	73
19	20
5	5
	3
$\hat{B}_{11} = 364$	$\hat{C}_{11} = 373$

Hence total enrolment

$$\hat{D}_{11} = 1000 + 364 = 1364$$

It is worthwhile to carry out the same verifying computation for the next column of the flow-diagram. We get [1]

Repetitions	Drop-outs	Total enrolment
113	139	460
58	71	123
22	28	33
8	9	9
	5	2
$\hat{B}_{22} = 201$	$\hat{C}_{22} = 252$	627
		+ 201
		$\hat{D}_{22} = 828$

Now there exists the possibility of introducing another sequence of important coefficients, linked to the data obtained by means of flow charts.

$$d_{j+1} = \frac{D_{j+1,j+1}}{D_{jj}} \; ; \quad j = 1,2,\ldots 4. \qquad (6)$$

Coefficient d_j will be called "conditional transition coefficient". [2]

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1 *The reader who is interested in a theoretical motivation of the simple method of calculating D_{jj} by means of the flow diagram (as it has been presented above) should address himself to [1] or [2]. It should be added, however, that we avoided presenting here some algebraic formulae not only to make this paper more easily understandable but first of all because these formulae hold true only when the rates are constant in time. In the § 11 we will consider a general case and then an effective solution can be obtained only through the flow-diagram concept.*

2 *Let P(II) be a probability for a pupil who entered the school to reach the grade 2.*

Hence $P(II) = P(II/I)\,P(I) = P(II/I)$ Since $P(I) = 1$

If now P(III), P(IV), P(V) are probabilities of a pupil reaching grades 3, 4 and 5 respectively, then we have

$$P(III) = P(III/II)\,P(II)$$
$$P(IV) = P(IV/III)\,P(III)$$
$$P(V) = P(V/IV)\,P(IV)$$

To simplify the notation we may put

$d_2 = P(II/I)$	Hence	$P(II) = d_2$
$d_3 = P(III/II)$		$P(III) = d_2\,d_3$
$d_4 = P(IV/III)$		$P(IV) = d_2\,d_3\,d_4$
$d_5 = P(V/IV)$		$P(V) = d_2\,d_3\,d_4\,d_5$

If we start now with the first year enrolment D_{11}, then the probable number of graduates say in the (-th year of schooling, that is D_{55} will fulfil the equality $E(D_{55}) = P(V)D_{11}$, where E stands for the operator of expected value. This simple formula will be exploited later on as a convenient tool for making predictions.

It follows from these two examples that the flow-diagram provides us with the possibility of obtaining a separation of enrolment, repetition and drop-out and therefore it allows us to examine the internal structure of these variables. On the other hand, this separation has no influence on the values of totals: B, C, D, since separation is an outcome and not a cause of these values. One can argue, therefore, about whether or not this separation is correct but the only fact which matters is the predictive power of the method considered, and this, as we will see later, is strong enough and deserves due attention.

10. PREDICTION I (Simplified Version)

The validity of any method should be verified by its potentiality to simulate facts of real life with a high degree of accuracy. This principle should also be applied to the method of describing educational wastage by the flow-diagram concept. For the sake of convenience this method will be called here "wastage prediction method" and coded as WPM.

It follows immediately from (6) that

$$D_{j+1,j+1} = d_{j+1} \times D_{jj}$$

Let $\hat{D}_{jj}$ denote an estimate of D_{jj} and $\hat{d}_j$ stand for the estimate of d_j.

Taking advantage of the WPM we can easily find

$$\hat{D}_{11} = 1364; \qquad \hat{d}_2 = \frac{828}{1364} = .607$$

$$\hat{D}_{22} = 828; \qquad \hat{d}_3 = \frac{449}{828} = .542$$

$$\hat{D}_{33} = 449; \qquad \hat{d}_4 = \frac{321}{449} = .715$$

$$\hat{D}_{44} = 321; \qquad \hat{d}_5 = \frac{260}{321} = .810$$

$$\hat{D}_{55} = 260$$

Hence if we know the initial value D_{11} and the sequence of indices $\hat{d}_1, \hat{d}_2, \hat{d}_3, \hat{d}_4$ then we can easily find the predicted value D_{jj}. We have

$$D^{*}_{22} = \hat{d}_2 D_{11}$$

$$D^{*}_{33} = \hat{d}_2\hat{d}_3 D_{11}$$

$$D^{*}_{44} = \hat{d}_2\hat{d}_3\hat{d}_4 D_{11}$$

$$D^{*}_{55} = \hat{d}_2\hat{d}_3\hat{d}_4\hat{d}_5 D_{11} \qquad (21)$$

In our case

D^{*}_{22} = 472,801 whereas the true value of D_{22} = 468,580

D^{*}_{33} = 256,258 whereas the true value of D_{33} = 269,718

D^{*}_{44} = 183,224 whereas the true value of D_{44} = 201,271

D^{*}_{55} = 148,411 whereas the true value of D_{55} = 166,440

(See Diagram 1, curve 1)

To throw still more light on the procedure under consideration some additional explanation concerning the symbols introduced may help.

All parameters which refer to the whole universe are denoted by conventional symbols, e.g. D_{ij}, a_j, b_j, c_j, d_j and so on. Parameters estimated by means of WPM are marked with one dash ($\hat{}$) as follows: $\hat{D}_{ij}$, $\hat{a}_j$, $\hat{b}_j$, $\hat{c}_j$, $\hat{d}_j$. When we intend to emphasize the fact that a value of a certain parameter plays not only the role of an estimate but simultaneously can be considered as a predictor of presently unknown value of an appropriate parameter, then we mark the relevant symbol with an asterisk (*).

As we have seen above, all predicted parameters D^{*}_{jj} were reasonable estimates of respective values of D_{jj}. To measure the degree of accuracy of this approximation we introduce the following variable :

$$\partial_j = \frac{\left| D^{*}_{jj} - D_{jj} \right|}{D_{jj}} \qquad j = 2,3,4,5.$$

In our case we get

j	∂_j
2	.009
3	.050
4	.090
5	.108

A decreasing trend in accuracy of prediction is by itself quite normal because it is fully understandable and theoretically well-founded that accuracy of prediction must decline

with a deeper penetration into unknown future. [1]

11. PREDICTION II (Developed Version)

In the § 10 a method of predicting educational wastage has been described. Prediction has been formulated on the basis of coefficients a_j, b_j, c_j which did not change over time. Now we are going to investigate whether or not the method could provide more accurate results if we applied, instead of a_j, b_j, c_j, new coefficients a_{ij}, b_{ij}, c_{ij} which would depend on time. This is simultaneously a crucial test for the validity of the assumption 8° in § 3.

We start with a table containing all values of coefficients a_{ij}, b_{ij}, c_{ij} (see Figure 8). At the bottom of the table arithmetic means, standard deviations and coefficients of variation have been computed. It is easy to notice that variation over time is everywhere rather small so that we could have been talking in terms of a linear trend with a mildly increasing slope over time.

Figure 8

Parameters	a_{ij}					b_{ij}					c_{ij}				
i \ j	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5
1	460	453	622	711	876	268	245	170	133	124	272	302	208	156	-
2	484	487	655	737	881	269	243	173	131	119	247	270	172	132	-
3	484	500	660	746	889	262	239	168	130	111	254	261	172	124	-
4	486	516	657	745	889	256	222	164	126	111	258	262	179	129	-
5	489	537	672	764	909	243	207	146	110	91	268	256	182	126	-
6	514	569	700	782	906	251	206	152	114	94	235	225	148	104	-
7	526	590	701	776	907	258	212	155	120	93	216	198	144	104	-
8	530	613	692	764	903	240	189	157	117	97	230	198	151	119	-
Mean	497	533	670	763	895	256	220	161	123	105	247	246	169	124	-
Standard deviation	.02	.05	.02	.02	.017	0	.017	0	0	0	014	.07	.017	.014	-
Coefficients of variation	.04	.09	.03	.03	.02	0	.08	0	0	0	06	.04	.1	.11	-

[1] *Especially if we take into account that we worked on the assumption of constant rates whereas actual rates (as can be seen in Figure 8) show an increasing trend. In such a case the method of prediction presented above should, as it is a case, lead to an underestimation of the true data.*

Here is the corresponding flow-diagram (see Figure 9).

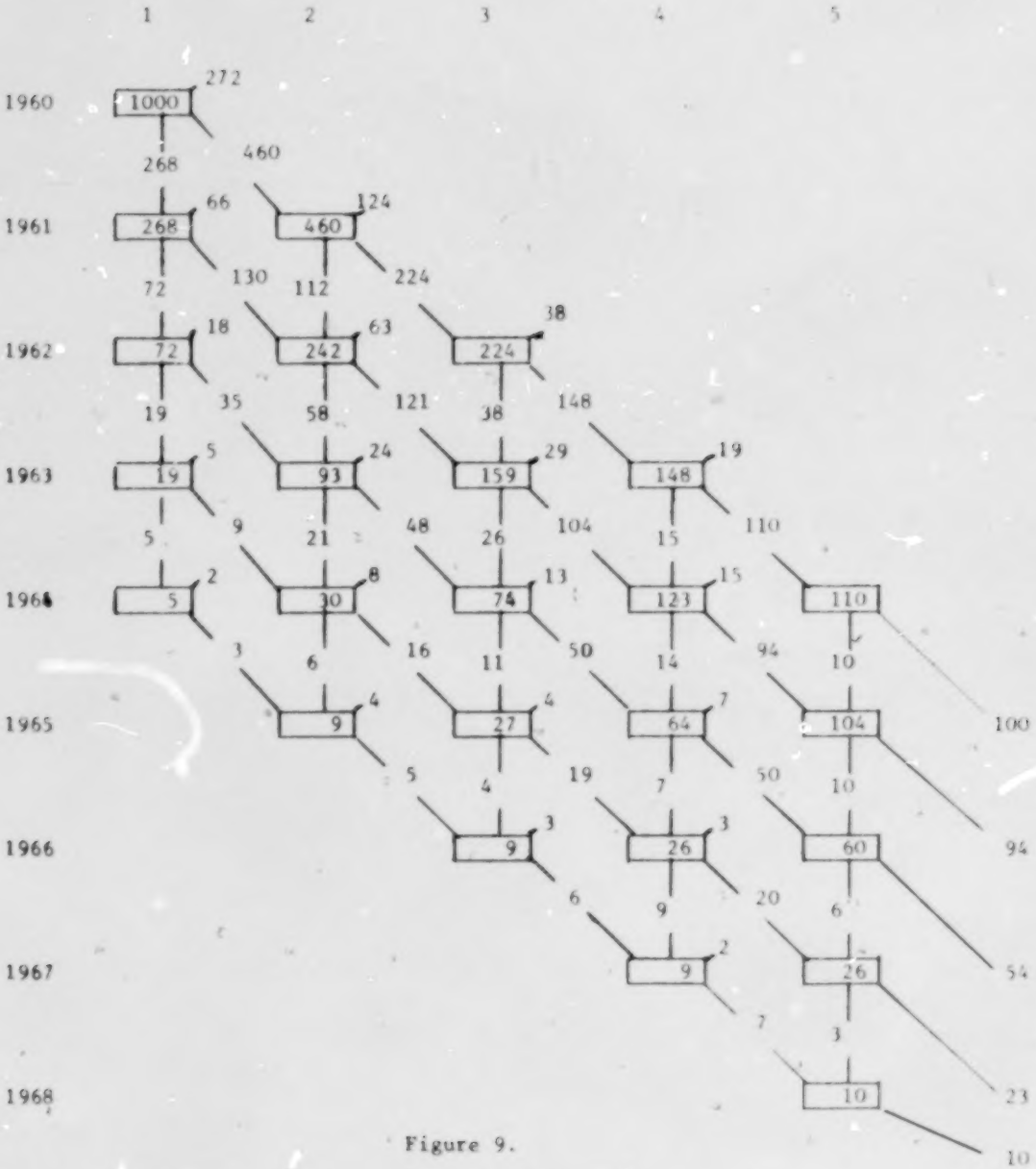

Figure 9.

It should be kept in mind that in practice we do not know in advance the rates as they are presented in Figure 8. Hence the second method could be applied for making projections given some various hypotheses about rates distributions have been made and we wished to examine different alternatives of possible future outcomes of the teaching process, as is the case when planners study different strategies in order to find the best one. The method II could also be of some help in the retrospective analysis of an educational process.

It is worth emphasizing, however, that even if we had known all rates as presented in Figure 8, the method itself would have produced a certain constant error which, in a case of rising trend of rates, would lead to an over-estimation of empirical data. This means that some further adjustment of projection would be highly desirable.

Applying the same procedure as was described above we get :

$\hat{D}_{11} = 1364$ and $\hat{d}_2 = \frac{834}{1364} = .611$

$\hat{D}_{22} = 834$ $\hat{d}_3 = \frac{493}{834} = .591$

$\hat{D}_{33} = 493$ $\hat{d}_4 = \frac{376}{493} = .763$

$\hat{D}_{44} = 376$ $\hat{d}_5 = \frac{310}{376} = .824$

$\hat{D}_{55} = 310$

Hence

$\hat{d}_2 D_{11} = .611 \times 778{,}914 = 415{,}916$

and in this way we get

$D^{*}_{22} = 475{,}916$ whereas the true value of $D_{22} = 468{,}580$

$D^{*}_{33} = 281{,}266$ whereas the true value of $D_{33} = 269{,}718$

$D^{*}_{44} = 214{,}606$ whereas the true value of $D_{44} = 201{,}271$

$D^{*}_{55} = 176{,}835$ whereas the true value of $D_{55} = 166{,}440$

(See Diagram 1, curve IV)

To measure accuracy of prediction we apply as before the measure δ_j

Here are the respective values of this parameter :

j	ϑ_j
2	0,016
3	0,043
4	0,067
5	0,089

12. PREDICTION III (Adjusted Version)

In this paragraph an attempt to improve the accuracy of prediction will be presented. To do this we apply the method I and the method II subsequently and make the arithmetic mean of values predicted be each of them. Since one tends to under- and the other to over-estimate the true data, the result of this simple adjustment will usually lead towards the improvement of prediction accuracy. So we have

$$D_{jj}^{III\,*} = \frac{D_{jj}^{I\,*} + D_{jj}^{II\,*}}{2}$$

where $D_{jj}^{I\,*}$, $D_{jj}^{II\,*}$, $D_{jj}^{III\,*}$ stand for predicted values of D_{jj} and are obtained by method I, method II and method III respectively.

After having done this we take the arithmetic means of the figures obtained. As it follows from direct computations we get:

D_{22}^{*} = 474,359 whereas the true value of D_{22} = 468,580

D_{33}^{*} = 268,762 whereas the true value of D_{33} = 269,718

D_{44}^{*} = 198,915 whereas the true value of D_{44} = 201,271

D_{55}^{*} = 167,623 whereas the true value of D_{55} = 166,440

To measure the accuracy of prediction we apply, as before, the coefficient ϑ_j

j	ϑ_j
2	.012
3	.003
4	.012
5	.007

This time we have succeeded in obtaining a very high degree of coherence between the true and forecasted values of D_{jj}. Especially worth emphasizing is the fact that the high accuracy coincides with the most removed moment of time i = 5. It is very important to examine which of the three methods described is the most reliable and to check if the method III could always deliver such astonishingly good results. To do this a lot of statistical work needs to be performed all based on original verified statistical data. A high degree of accuracy of prediction which results from application of WPM supports very strongly the method itself and gives one good reason to believe that the WPM describes accurately and adequately all the crucial aspects of educational wastage. On the other hand, however, when we apply the flow chart as a tool of prediction we should always keep strongly in mind that it has been shown in the paper, both m thod I and II will produce a constant error or under- or over-estimation if there is a trend in rates data. Since we never know in advance the rates, we will always be faced with three possible ways of handling the prediction procedure : 1° apply the method I, starting with the last available grades; 2° introduce some method of prediction change in rates and then apply both methods I and II; 3° put aside the whole flow-chart concept and develop an entirely new approach to the problem of prediction in the field of education.

13. FIT-HIT ANALYSIS

Any educational process can be looked upon as an elimination procedure. It means that not all pupils will succeed in passing from one grade to another: some will fail either in the sense that they will have to repeat the same grade or will leave the school. Of course the probability of success depends to a large extent on the pupil's endurance, ability and brightness, good health, will to succeed and many other factors connected directly with the pupil himself. If a pupil possesses all the appropriate merits, developed to such an extent that they enable him to pass from one grade to another, we will say that he is fit for studying the next grade. It is also true that the probability of the student's success depends on the professor's ability to distinguish between a good and a bad student, on his experience and intuition which should help him to break the group of students into two sub-groups: one containing those and only those students who are fit for studying at the next grade and the other embracing all the remaining ones. If the teacher's judgement is correct then we will call his verdict a hit. Hence hit means that a collective verdict of the teaching staff is positive when a pupil is fit and negative if he is not.

Let f denote a probability that a given student is fit and h stand for a probability of teacher's hit. We are going to show that prbabilities f and h can be found on the basis of existing statistical data. Let us find a probability of students' successful passing from the first to the second grade during the academic session in 1963. We have

$$fh + (1 - f)(1 - h) = a_1$$

In exactly the same way we can find the probability of students again successfully passing, this time from the second to the third grade. We get

$$fh^2 + (1 - f)(1 - h)^2 = a_1 \times a_2$$

Hence we arrive at the following set of two equations:

$$XY + (1 - X)(1 - Y) = p$$

$$XY^2 + (1 - X)(1 - Y)^2 = q$$

where

$$p = a_1^{(1963)} = .486$$

$$q = a_1^{(1963)} \, a_2^{(1964)} = .486 \times .537 = .2609$$

The solution [1] is given by the following formulae :

$$X = \frac{Y + p - 1}{2Y - 1} ;$$

$$Y_{1,2} = \frac{1 \pm \sqrt{1 - 4p(1 - q/p)}}{2}$$

In our case

$$X_1 = .5443 \qquad Y_1 = .3419$$

$$X_2 = .4557 \qquad Y_2 = .6581$$

We choose the second pair of solutions and obtain

Year 1963, grade 1,2 : $f_1 = .4557$; $h_1 = .6581$;

In exactly the same way we obtain also :

Year 1965, grade 2,3 : $f_2 = .5620$; $h_2 = .7982$;

Year 1965, grade 3,4 : $f_3 = .8276$; $h_3 = .8053$;

Year 1966, grade 4,5 : $f_4 = .8301$; $h_4 = .9180$

---

[1] *I am pleased to express my gratitude to Miss A. Kania for her assistance in performing the numerical computations.*

"Fit - Hit" analysis has various possible applications. We will show how it could be exploited for checking assumption 7° in § 3. As we recall, the WPM could have been exposed to serious criticism if we had not been able to establish that this assumption does not contradict the real facts.

Let

$P_1(A/O)$ denote the probability that a student who has entered the first grade will pass to the second grade, and

let

$P_1(A/B, \bar{C})$, $P_2(A/B, C)$ be probabilities that a student who has repeated the first grade will pass to the next grade given that drop-out is respectively being neglected or taken into account.

Hence if

Year: 1963 / grade: I — $f = .4557$, $(1 - f) = .5443$; $h = .6581$, $(1 - h) = .3419$; $c = .258$

then

$$P_1(A/O) = fh + (1 - f)(1 - h) = .4858$$

$$P_1(A/B, C) = \frac{f(1-h)h + (1-f)h(1-h)}{1 - P_1(A/O)} = \frac{h(1-h)}{1 - P_1(A/O)} = .4375$$

$$P_1(A/B, \bar{C}) = \frac{h(1-h)}{1-C-P_1(A/O)} = .8782$$

This means that the probability of successful passing from one grade to another for a repeater and non-repeater depends to a large extent on whether we take or do not take into account all those students who drop out. If we do, then this probability for a repeater is slightly greater than for a non-repeater. If, however, we do not, then exactly the opposite is true.

Here are all the probabilities which correspond to the years 1963 - 1966 and to the grades 1, 2, 3, 4.

Year: 1963
grade I

$f = .4557$ $\quad h = .6581$

$(1 - f) = .5443$ $\quad (1 - h) = .3419$

$c = .258$

$P_1(A/O) = .4858$

$P_1(A/B, C) = .4375$

$P_1(A/B, \bar{C}) = .8782$

Year: 1963
grade II

$f = .5282$ $\quad h = .7842$

$(1 - f) = .4718$ $\quad (1 - h) = .2158$

$c = .262$

$P_2(A/O) = .5160$

$P_2(A/B, C) = .3495$

$P_2(A/B, \bar{C}) = .7621$

Year: 1963
grade III

$f = .7548$ $\quad h = .8081$

$(1 - f) = .2452$ $\quad (1 - h) = .1919$

$c = .179$

$P_3(A/O) = .6539$

$P_3(A/B, C) = .4478$

$P_3(A/B, \bar{C}) = .9275$

Year: 1963
grade IV

$f = .7870$ $\quad h = .9269$

$(1 - f) = .2130$ $\quad (1 - h) = .0731$

$c = .129$

$P_4(A/O) = .7449$

$P_4(A/B, C) = .2653$

$P_4(A/B, \bar{C}) = .5368$

Year: 1964
grade I

$f = .4722$ $\quad h = .6981$

$(1 - f) = .5278$ $\quad (1 - h) = .3019$

$c = .268$

$P_1(A/O) = .4074$

$P_1(A/B, C) = .3555$

$P_1(A/B, \bar{C}) = .6491$

Year: 1964
grade II

$f = .5620$ $\quad h = .7982$

$(1 - f) = .4380$ $\quad (1 - h) = .2018$

$c = .256$

$P_2(A/O) = .5368$

$P_2(A/B, C) = .3475$

$P_2(A/B, \bar{C}) = .7770$

Year: 1964
grade III

$f = .7673$ $\quad h = .8217$

$(1 - f) = .2327$ $\quad (1 - h) = .1783$

$c = .182$

$P_3(A/O) = .6718$

$P_3(A/B, C) = .4463$

$P_3(A/B, \bar{C}) = 1.000$

Year: 1964
grade IV

$f = .8127$ $\quad h = .9221$

$(1 - f) = .1873$ $\quad (1 - h) = .0779$

$c = .126$

$P_4(A/O) = .7638$

$P_4(A/B, C) = .3039$

$P_4(A/B, \bar{C}) = .6515$

Year: 1965
grade I

$f = .5353$ $\quad h = .6981$ $\quad c = .235$

$(1 - f) = .4647$ $\quad (1 - h) = .3019$

$P_1(A/O) = .5138$

$P_1(A/B, C) = .4333$

$P_1(A/B, \bar{C}) = .8387$

Year: 1965
grade II

$f = .6221$ $\quad h = .7826$ $\quad c = .225$

$(1 - f) = .3779$ $\quad (1 - h) = .2174$

$P_2(A/O) = .5689$

$P_2(A/B, C) = .3945$

$P_2(A/B, \bar{C}) = .8253$

Year: 1965
grade III

$f = .8276$ $\quad h = .8053$ $\quad c = .148$

$(1 - f) = .1724$ $\quad (1 - h) = .1947$

$P_3(A/O) = .6999$

$P_3(A/B, C) = .5221$

$P_3(A/B, \bar{C}) = 1.000$

Year: 1965
grade IV

$f = .8349$ $\quad h = .9210$ $\quad c = .104$

$(1 - f) = .1651$ $\quad (1 - h) = .0790$

$P_4(A/O) = .7819$

$P_4(A/B, C) = .3333$

$P_4(A/B, \bar{C}) = .6319$

Year: 1966, grade I: $f = .5603$, $(1 - f) = .4397$, $h = .7155$, $(1 - h) = .2845$, $c = .216$

$P_1(A/O) = .5258$

$P_1(A/B, C) = .4291$

$P_1(A/B, \bar{C}) = .7881$

Year: 1966, grade II: $f = .6722$, $(1 - f) = .3278$, $h = .7613$, $(1 - h) = .2387$, $c = .158$

$P_2(A/O) = .5899$

$P_2(A/B, C) = .4430$

$P_2(A/B, \bar{C}) = .7207$

Year: 1966, grade III: $f = .8456$, $(1 - f) = .1544$, $h = .7908$, $(1 - h) = .2092$, $c = .144$

$P_3(A/O) = .7610$

$P_3(A/B, C) = ..5531$

$P_3(A/B, \bar{C}) = 1.000$

Year: 1966, grade IV: $f = .8301$, $(1 - f) = .1699$, $h = .9180$, $(1 - h) = .0820$, $c = .104$

$P_4(A/O) = .7759$

$P_4(A/B, C) = .3355$

$P_4(A/B, \bar{C}) = .6261$

According to the "fit - hit" analysis and from all comparisons between consecutive grades carried out over time, we are entitled to draw the following conclusion : the probability of passing from one year to another for non-repeaters is smaller than for repeaters if drop-outs are taken into account and the other way round if drop-outs are not taken into account. This probability is, however, always large in the last grades of the schooling process. Since the discrepancies are of medium size they do not contradict convincingly the assumption 8° in § 3.

D I A G R A M 1

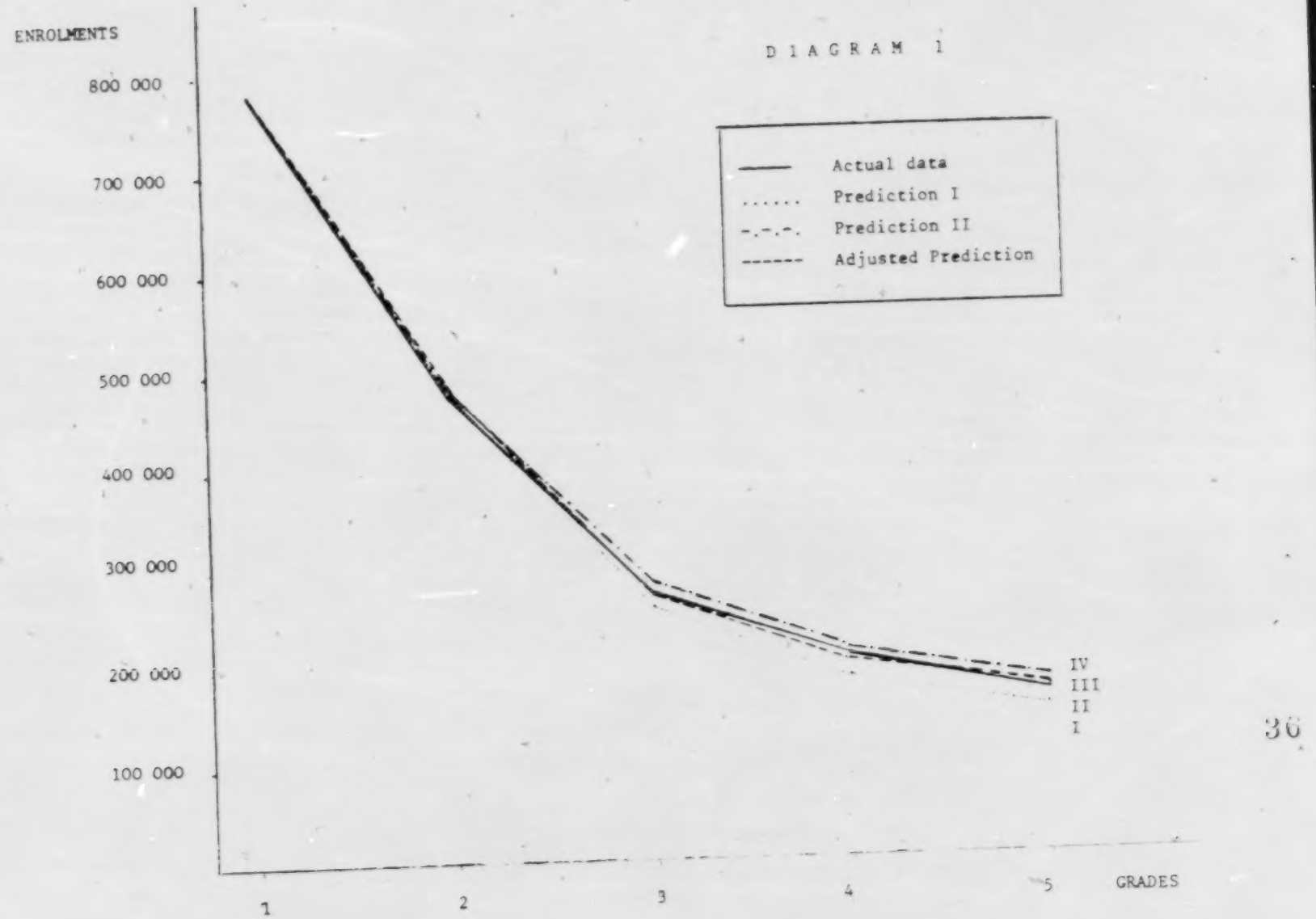
ENROLMENTS
800 000
700 000
600 000
500 000
400 000
300 000
200 000
100 000
Actual data
Prediction I
Prediction II
Adjusted Prediction
IV
III
II
I
1
2
3
4
5
GRADES

Table 1

Year and category	Sex of pupil	First level	grade 1	grade 2	grade 3	grade 4	grade 5
1960	Both sexes	1690361	778914	448744	224197	142259	96247
Total enrolment	Female	842619	380506	226467	113745	71918	49995
of which repeaters	Both sexes	367808	200174	106024	34300	17039	10271
	Female	178992	94698	53387	17125	8559	5225
1961	Both sexes	1791813	810441	468580	241298	158398	113096
Total enrolment	Female	888377	394948	235166	121434	80118	56711
of which repeaters	Both sexes	388070	209045	110051	38177	18861	11936
	Female	187332	98435	55162	18664	9175	5896
1962	Both sexes	1948772	863602	506274	269718	178909	130269
Total enrolment	Female	970518	424491	254392	136132	89600	65903
of which repeaters	Both sexes	408088	218029	114066	41720	20778	13495
	Female	198695	103941	57161	20555	10234	6804
1963	Both sexes	2096408	910099	539030	298145	201271	147863
Total enrolment	Female	1040397	443372	272271	149872	100559	74323
of which repeaters	Both sexes	430375	226221	121231	45193	23311	14419
	Female	206237	106470	59977	22140	10969	6681

Year and category	Sex of pupil	First level	grade 1	grade 2	grade 3	grade 4	grade 5
1964	Both sexes	2213423	936972	561748	327012	221251	166440
Total enrolment	Female	1105380	461698	283875	164622	111720	83465
of which repeaters	Both sexes	443711	233350	119586	48899	25440	16436
	Female	214500	110728	60060	23584	12480	7648
1965	Both sexes	2274014	922056	574162	349324	244309	184163
Total enrolment	Female	1146168	460003	292535	177329	123645	92656
of which repeaters	Both sexes	431289	227672	116264	47889	24380	15084
	Female	210869	109341	58384	23664	12080	7400
1966	Both sexes	2402030	949341	592152	379939	272547	208060
Total enrolment	Female	1206473	471666	300294	192750	136990	104773
of which repeaters	Both sexes	447537	231247	118252	52955	27851	17232
	Female	219526	110439	60119	26802	13867	8299
1967	Both sexes	2586288	1010967	628069	408427	298922	230833
Total enrolment	Female	1296105	504026	318692	206522	150761	116204
of which repeaters	Both sexes	482400	246532	125036	58811	32592	19420
	Female	243601	116980	62610	29335	16303	9373
1968	Both sexes	2733432	1056066	659476	449154	317862	250874
Total enrolment	Female	1369499	169526	518988	344200	228047	160923
of which repeaters	Both sexes	484884	244402	118862	64053	35112	22455
	Female	234608	116179	59428	31248	17385	10368

Table 2

RATES		M + F				
		1	2	3	4	5
1960	Repeaters	.268	.245	.170	.133	.124
	Promoted	.460	.453	.622	.711	.876
	Drop-out	.272	.302	.208	.156	
1961	Repeaters	.269	.243	.173	.131	.199
	Promoted	.484	.487	.655	.737	.881
	Drop-out	.247	.270	.172	.132	
1962	Repeaters	.262	.239	.168	.130	.111
	Promoted	.484	.500	.660	.746	.889
	Drop-out	.254	.261	.124	.124	
1963	Repeaters	.256	.222	.164	.126	.111
	Promoted	.486	.516	.657	.745	.889
	Drop-out	.258	.262	.176	.129	
1964	Repeaters	.243	.207	.146	.110	.091
	Promoted	.486	.537	.672	.764	.909
	Drop-out	.268	.256	.182	.126	
1965	Repeaters	.251	.206	.152	.114	.094
	Promoted	.514	.569	.700	.782	.906
	Drop-out	.235	.225	.148	.104	
1966	Repeaters	.258	.212	.155	.120	.093
	Promoted	.526	.590	.701	.776	.907
	Drop-out	.216	.158	.144	.104	
1967	Repeaters	.240	.189	.157	.117	.097
	Promoted	.530	.613	.692	.764	.903
	Drop-out	.230	.198	.151	.119	

RATES		M				
		1	2	3	4	5
1960	Repeaters	.278	.247	.177	.138	.131
	Promoted	.448	.451	.621	.716	.869
	Drop-out	.274	.302	.202	.146	
1961	Repeaters	.275	.244	.177	.135	.119
	Promoted	.469	.482	.657	.737	.881
	Drop-out	.256	.274	.166	.128	
1962	Repeaters	.273	.243	.173	.138	.120
	Promoted	.468	.497	.662	.737	.880
	Drop-out	.259	.260	.165	.125	
1963	Repeaters	.263	.223	.171	.129	.119
	Promoted	.468	.514	.651	.737	.881
	Drop-out	.269	.263	.178	.134	
1964	Repeaters	.249	.208	.149	.112	.093
	Promoted	.471	.532	.667	.765	.907
	Drop-out	.280	.260	.184	.123	
1965	Repeaters	.261	.206	.153	.116	.098
	Promoted	.506	.571	.707	.782	.902
	Drop-out	.233	.223	.140	.102	
1966	Repeaters	.268	.214	.157	.120	.097
	Promoted	.511	.591	.705	.771	.903
	Drop-out	.221	195	.138	.109	
1967	Repeaters	.249	.192	.163	.120	.105
	Promoted	.515	.608	.689	.753	.895
	Drop-out	.236	.200	.148	.129	

RATES		F				
		1	2	3	4	5
1960	Repeaters	.259	.244	.164	.127	.118
	Promoted	.473	.454	.624	.706	.882
	Drop-out	.268	.302	.212	.167	
1961	Repeaters	.263	.243	.169	.128	.120
	Promoted	.499	.491	.654	.738	.880
	Drop-out	.238	.266	.177	.134	
1962	Repeaters	.251	.236	.163	.122	.101
	Promoted	.500	.502	.658	.755	.899
	Drop-out	.249	.262	.179	.123	
1963	Repeaters	.250	.221	.157	.124	.103
	Promoted	.505	.518	.662	.754	.897
	Drop-out	.245	.261	.181	.122	
1964	Repeaters	.237	.206	.144	.108	.089
	Promoted	.507	.541	.677	.763	.911
	Drop-out	.256	.253	.179	.129	
1965	Repeaters	.240	.206	.151	.112	.090
	Promoted	.522	.568	.694	.781	.910
	Drop-out	.238	.226	.155	.107	
1966	Repeaters	.247	.208	.152	.199	.089
	Promoted	.542	.590	.698	.780	.911
	Drop-out	.211	.202	.150	.101	
1967	Repeaters	.231	.187	.151	.115	.189
	Promoted	.546	.618	.695	.775	.911
	Drop-out	.223	.195	.154	.110	

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SOME OBSERVATIONS ON PROFESSOR HELLWIG'S TEXT

D. Blot

My observations will bear on two points : firstly, pages 14 - 24 (Standard Model of Educational Wastage) and secondly, more briefly, pages 24 onwards (Fit-Hit Analysis).

I. Standard Model of Educational Wastage

1. Let us start with an analysis of the evolution of a cohort as indicated on page 14 taking the rates of repetition (b_i), promotion (a_i) and drop-outs (c_i) to be constant. Taking N to denote the original size of the cohort, the following relations can be demonstrated 1/ :

$$D_{11} = \frac{N}{1 - b_1}$$

$$D_{22} = \frac{a_1 N}{(1 - b_1)(1 - b_2)}$$

$$D_{33} = \frac{a_1 a_2 N}{(1 - b_1)(1 - b_2)(1 - b_3)}, \text{ etc}$$

and, in general :

$$D_{jj} = \frac{a_1 a_2 \ldots a_{j-1} N}{(1 - b_1)(1 - b_2) \ldots (1 - b_j)} \quad 1/$$

It should be explained that these formulae apply only to an isolated cohort. I shall however revert to this point later, though noting it here, since the letter D is used, throughout Professor Hellwig's text, to denote both a cohort and enrolments actually observed.

...

1/ For the demonstration of this, see for example Annex D to : Education, Human Resources and Development in Argentina : methodological problems and statistical data, OECD, 1968.

If we replace a_j and b_j by the figures on page 16 we find, assuming N = 1 000 000 for example,

D_{11} = 1 366 120

D_{22} = 832 338

D_{33} = 454 276

D_{44} = 325 905

D_{55} = 264 519

It is shown further that the drop-outs at the end of each grade - let us designate them by W_{jj} - are given by :

$$W_{jj} = c_j D_{jj}$$

or, in the example :

W_{11} = 371 585

W_{22} = 251 366

W_{33} = 94 489

W_{44} = 50 841

W_{55} = 231 719

the total output being evidently equal to 1 000 000.

It will be noted that the values of D calculated above differ slightly from those given on page 18 of the text. The point is that the latter are calculated from the flow-

diagram and that a diagram of this type, at least if we wish to make it sufficiently simple, contains errors of approximation 1/.

These are perhaps very small and unnecessarily pernickety points. But if the values of δ (cf. p. 19) are calculated from the preceding results, the figures obtained differ substantially from those contained in the text. We obtain in fact :

0.013 instead of 0.009
0.040 instead of 0.050
0.077 instead of 0.090
0.084 instead of 0.108

The upshot of this is that, if we are concerned with what finally happens to the cohort, it is preferable to use the above formulae directly rather than the flow-diagram, the main purpose of which is to show simply and clearly the evolution of the cohort year by year.

Professor Hellwig, in a note on page 17, refers to these formulae. He says that he refrained from using them partly in order to make the paper more easily understandable and above all because they hold true only when the rates are constant in time. I am perfectly prepared to accept the first argument, but the second does not seem to me to be convincing : the calculations on page 17 are in fact likewise based on the assumption that the rates are constant, since it is this assumption that makes it possible for the flow-diagram used to be drawn up.

2. I come now to a more fundamental objection. Professor Hellwig, throughout his paper, assumes that the results obtained for one cohort apply as they stand to enrolments actually observed. I therefore propose to show that this assumption is true only in a very special case, which is not that of Colombia, taken as our example.

In the following, I shall take $S_j(t)$ to denote the enrolment for the j-th grade in school year t (for example S_1 (1960) = 778 914), I shall keep D_j to designate the results obtained for one cohort, deleting the second letter as necessary to simplify the presentation. In these conditions we have, according to page 18 :

$$D_{j+1} = d_{j+1} D_j \qquad (2)$$

and the calculations on pages 18 and 19 imply that :

$$S_{j+1}(t+1) = d_{j+1} S_j(t) \qquad (3)$$

i.e. :

$$S_j(t+j-1) = d_2 d_3 \cdots d_j S_1(t).$$

Formula (2) which applies to a cohort is true by definition, but I shall show that formula (3) applied to the observed enrolments is true only in a particular case.

1/ In addition it seems that, in working out this diagram, allowance was made for at most 4 repetitions in each grade. Otherwise, it would be incomprehensible, for example, why the 10 second-year (grade) students in 1965, indicated on page 15 do not produce $10 \times 0.245 = 2$ repeaters in 1966.

...

If the rates of repetition and promotion are assumed to be constant (which is the basic hypothesis here), on what does the enrolment for each grade depend ? It depends on the enrolment for the preceding school year and on the number of new pupils, all of whom, for the sake of simplicity, we shall assume enter the first grade. Taking N (t) to be the enrolment of new pupils at the beginning of school year t, we shall have the following system of equations :

$$S_1(t+1) = b_1 S_1(t) + N(t+1)$$

for the first grade and :

$$S_j(t+1) = b_j S_j(t) + a_{j-1} S_{j-1}(t)$$

for subsequent grades. Let us suppose, for the sake of illustration, that the enrolment of new pupils increases by g % per year, we shall have [1] :

$$N(t) = (1+g)^t N(o)$$

The solution of the system of equations is then :

$$S_1(t) = k_{11} b_1^t + \frac{1}{1+g-b_1} (1+g)^{t+1} N(o)$$

$$S_j(t) = k_{1j} b_1^t + k_{2j} b_2^t + \ldots + k_{jj} b_j^t + \frac{a_1 a_2 \ldots a_{j-1}}{(1+g-b_1)(1+g-b_2)\ldots(1+g-b_j)} (1+g)^{t+1} N(o)$$

in which the coefficients k depend on the initial distribution of the enrolments for the year t = o. When the exponent t increases, the terms containing b_j^t approach zero, since b_j is by definition lower than 1, and, at the limit, we have :

$$\lim S_j(t) = \frac{a_1 a_2 \ldots a_{j-1}}{(1+g-b_1)(1+g-b_2)\ldots(1+g-b_j)} (1+g)^{t+1} N(o)$$

In these conditions, it is calculated that the coefficient d_{j+1} of the expression (3) is equal to :

$$\frac{(1+g)\, a_j}{1+g-b_{j+1}}$$

whilst that of the expression (2), calculated from the formulae given at the beginning of this note, is equal to :

$$\frac{a_j}{1-b_{j+1}}$$

1/ Cf. the survey on Argentina, Annex D already quoted, where we shall also find the hypotheses necessary for the model, which are not given here.

In the final analysis, this last equality, which is implicitly used by Professor Hellwig, is exact only if g = 0, i.e. when the annual enrolment of new pupils remains constant from year to year. As a corollary, the analysis made of a cohort can only be applied as it stands to enrolments actually observed if the annual enrolment of new entrants is assumed to be constant.

Since this is not true in the case of Colombia - as can be seen from Table 1 of the Appendix - the validity of the calculations on page 1 becomes questionable. In addition, the calculations themselves appear to contain a contradiction. If the rates of repetition and promotion for 1960 are known - which is the basic hypothesis - then the enrolment for 1960 and 1961, from which these rates are calculated, is likewise known. It is therefore contradictory to find for the second grade in 1961 an enrolment figure which differs from the real figure (472 801 instead of 468 580), since the real figure is known by hypothesis.

3. There are however other ways of testing the validity of the model. Let us suppose that, after 1961, the last year for which the figures are by hypothesis available, the enrolment of new pupils increases by 4 % per year (i.e. about the average rate from 1960 to 1968) whilst the repetition and promotion rates remain unchanged, at the 1960 level. We then have all the necessary data for calculating the enrolment for each grade for one school year after another.

The results obtained are shown below, down to the year 1968, together with percentage breakdown of total enrolment by grades. From 1964 onwards, these percentages remain unchanged and moreover it will be noticed that during the last school years it is not only the intake of new entrants that increases by 4 % per year but the enrolment for each grade. These two facts are not accidental, and it can easily be shown that this must be so.

(thousands)

	Enrolment of new entrants	1st grade	2nd grade	3rd grade	4th grade	5th grade	TOTAL
1961	601.4	810.4	468.6	241.3	158.4	113.1	1,791.8
1962	625.5	842.7	487.6	253.3	171.2	126.6	1,881.4
1963	650.5	876.3	507.1	264.0	180.4	137.4	1,965.2
1964	676.5	911.3	527.3	274.6	188.2	145.3	2,046.7
1965	703.6	947.8	548.3	285.6	195.8	151.8	2,129.4
1966	731.7	985.7	570.4	297.0	203.6	158.0	2,214.7
1967	761.0	1,025.2	593.1	308.9	211.8	164.4	2,303.4
1968	791.4	1,066.2	616.9	321.2	220.3	171.0	2,395.6

(%)

	1st grade	2nd grade	3rd grade	4th grade	5th grade	TOTAL
1961	45.2	26.2	13.5	8.8	6.3	100.0
1962	44.8	25.9	13.5	9.1	6.7	100.0
1963	44.6	25.8	13.4	9.2	7.0	100.0
1964	44.5	25.8	13.4	9.2	7.1	100.0
1965	44.5	25.8	13.4	9.2	7.1	100.0

If we compare these theoretical enrolment figures with the actual figures given in the annex, it is clear at a glance that there are substantial differences. Would the results have been better if the rate chosen for the increase in the number of new entrants had been other than 4 % ? I shall reply to this question in a moment. In any case, the enrolment figures shown above enclosed in a ladder diagram are independent of the rate of increase selected, and the percentage differences in relation to the actual figures are as follows :

	1st grade	2nd grade	3rd grade	4th grade	5th grade
1961	0	0	0	0	0
1962		-3.7	-6.1	-4.3	-2.8
1963			-11.4	-10.4	-7.1
1964				-15.0	-12.7
1965					-17.6

It will be seen that the differences, all in the same direction, are fairly large and, above all, that they increase.

Is it then possible to improve the position by modifying the rate of increase of the new entrants, at least for the enrolments affected by that rate ? This is in fact impossible. It can be shown that, always assuming that the rates of repetition and promotion are constant, the limiting theoretical distribution of enrolments, for different rates of increase, would be as follows :

...

Annual rate of increase	Theoretical distribution %					
	1st grade	2nd grade	3rd grade	4th grade	5th grade	TOTAL
0	42.1	25.7	14.0	10.0	8.2	100.0
3.0	43.9	25.7	13.6	9.4	7.4	100.0
3.5	44.2	25.7	13.5	9.3	7.3	100.0
4.0	44.5	25.8	13.4	9.2	7.1	100.0
4.5	44.8	25.8	13.3	9.1	7.0	100.0

whereas the actual distribution was :

	1st grade	2nd grade	3rd grade	4th grade	5th grade	TOTAL
1960	46.1	26.5	13.3	8.4	5.7	100.0
1961	45.2	26.2	13.5	8.8	6.3	100.0
1962	44.3	26.0	13.8	9.2	6.7	100.0
1963	43.4	25.7	14.2	9.6	7.1	100.0
1964	42.3	25.4	14.8	10.0	7.5	100.0
1965	40.5	25.2	15.4	10.7	8.1	99.9
1966	39.5	24.7	15.8	11.3	8.7	100.0
1967	39.4	24.3	15.8	11.6	8.9	100.0
1968	38.6	24.1	16.4	11.6	9.2	99.9

It is clear that no likely rate of increase will produce distribution figures of this order. As a result, the hypothesis according to which repetition and promotion rates are constant is not tenable in the specific case taken as an example.

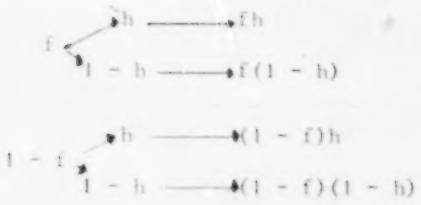
From page 17 onwards, Professor Hellwig abandons the hypothesis that these rates are constant ; but it seems to me that the above objections continue to apply to a no lesser degree. I note for example that the calculated value of S_2 (1961) is 475 919, whereas the actual value is 468 510. In fact, since the repetition and promotion rates are all given by hypothesis, the calculated figures should coincide exactly with the actual figures. The fact that this is not so shows, in my opinion, that there is a built-in contradiction in the method used.

...

II. Fit-Hit Analysis

This part of the text is very different from the preceding ones, and I shall confine myself to a few comments only.

Pupils can be either "fit" (probability : f) or not "fit" (probability : 1 - f). In either case, they can be considered "hit" (probability : h) or not (probability : 1 - h). We thus have the following "probability tree" :



Thus the probability of being "fit" is independent of the probability of being "hit", a hypothesis which seems to me to be highly unlikely.

On page 21, we find the two following equations :

$$X\,Y + (1 - X)\,(1 - Y) = a_1$$

$$X\,Y^2 + (1 - X)\,(1 - Y)^2 = a_1\,a_2$$

This system has two pairs of solutions, but Professor Hellwig retains one only, without explaining the reasons for his choice, though it would be particularly useful if he did so in that the other pair of solutions is very different and appears at first glance to be equally probable.

Then again, why confine ourselves to two equations ? We could equally well write :

$$X\,Y^3 + (1 - X)\,(1 - Y)^3 = a_1\,a_2\,a_3$$

$$X\,Y^4 + (1 - X)\,(1 - Y)^4 = a_1\,a_2\,a_3\,a_4$$

$$X\,Y^5 + (1 - X)\,(1 - Y)^5 = a_1\,a_2\,a_3\,a_4\,a_5$$

But in this case the system would be indeterminate, and it would be necessary, in order to solve it, to use supplementary hypotheses, which would certainly produce yet other solutions.

THE QUANTIFICATION OF SCHOOL EVENTS RELATED TO EDUCATIONAL WASTAGE

A. Brimer

The ingenuity that is possible in the manipulation of simple enrolment data, grade by grade and year by year, to secure estimates of the gross behaviour of education systems, has challenged statisticians to construct models that either ignore the erratic quality of the data or compensate for its intrinsic error. Better data has been desired but usually better implies "more reliable" data, "more frequently". Ideal data is regarded as "true cohort" data derived from immaculately kept longitudinal records. The events to be recorded by this method are nevertheless, movements of individual pupils in and out of the system and between grades and years. Rarely has the triviality of the events themselves been called in question. International concern over the problem of "educational wastage" reached its peak with the 32nd session of the International Conference on Education in 1970, but the best estimates of wastage then available could only reflect the non-profitable absorption by pupils of educational resources as inferred from such movement of pupils.

Just as to have a conception of error implies a prior notion of true value, so the conception of wastage implies a prior notion of perfect efficiency. If one is to apply the notion of perfect efficiency to the events represented by pupil movements, it would mean a state of affairs whereby all pupils entering the school at the beginning of a cycle moved up through the system one year for each grade, without repetition or drop-out, to graduate from the cycle in the minimum time. Were children identical in response to educational provision, this would be achieved without loss of educational standards. But since real children differ widely in their response to provision, it could only be achieved by employing differential grade standards throughout and at the end of the cycle. Since some children would apparently never reach the standard of the final grade, however long they spent in school, it is clear that no system seriously supposes that it has the same intentions of achievement for all children. It follows that even the theoretical perfection of a system must acknowledge educational events rather than pupil movement. Even if drop-outs are excluded from consideration, the average number of years that a pupil requires to move through a system to graduation must be greater than the number of grades. To regard a system as inefficient because it adapts to differential rates of learning amongst the pupils is at best inaccurate and at worst misleading.

Bloom (1969) referring to Carroll's (1963) paper and deriving a learning strategy for skill mastery, poses the following concepts :

> "If students are normally distributed with respect to aptitude for some subject (mathematics, science, literature, history, etc.) and all the students are provided with exactly the same instruction (same in terms of amount of instruction, quality of instruction, and time available for learning), the end result will be a normal

distribution on an appropriate measure of achievement. Furthermore, the relationship between aptitude and achievement will be relatively high (correlations of + 0.70 or higher are to be expected if the aptitude and achievement measures are valid and reliable). Conversely, if the students are normally distributed with respect to aptitude, but the kind and quality of instruction and the amount of time available for learning are made appropriate to the characteristics and needs of each student, the majority of students may be expected to achieve mastery of the subject. And, the relationship between aptitude and achievement should approach zero".

From this it can be seen that Carroll has put forward a notion of aptitude for learning in which it is regarded as the time required by the learner to attain mastery of a task. The assumption is that all students can attain mastery of a curriculum if they are given enough time, and conversely, that if time is constant, there will be differential levels of achievement. The remainder of the model is of considerable interest to those concerned with the diagnostic evaluation of educational systems, but the present paper will restrict its use of the model to the central concept.

The concept is most obviously applicable to educational systems of the type we have so far discussed - systems which provide courses of study of one year's duration, culminating in assessment and, in the following year, promotion to the next grade or repetition of the course or drop-out. Systems of this type we might call 'fixed level' systems. The concept is not so obviously applicable to systems which promote automatically at the end of each year and which exclude drop-out. Such systems permit a continuous distribution of pupil achievement levels within each year of the cohort, and may, since the duration of passage through the cycle is the same for all children, be called 'fixed duration' systems.

Implicit in the concept, is the idea that inefficiency in an educational system is represented by failure to ensure that each pupil learns at the rate of which he is capable and ineffectiveness by its failure to provide sufficient time for him to attain mastery of the curriculum. This poses certain problems for a mode of assessing system efficiency which relies upon "excess input of pupil years" as a measure of wastage. The excess is computed by subtracting the minimal number of pupil years, required for the output of graduates from the cohort, from the total input of pupil years. In other words, all input which is greater than one year per pupil per grade is regarded as wasteful, whereas, broadly speaking, the Carroll model would regard some input of this kind as evidence of profitable investment in a system in providing sufficient time for differential learning rates. Likewise, all years spent in a cycle by pupils who drop out before completing it are regarded as wasteful, whereas the Carroll model would regard the system as ineffective in that it failed to provide sufficient time for learning through its inadequate holding power.

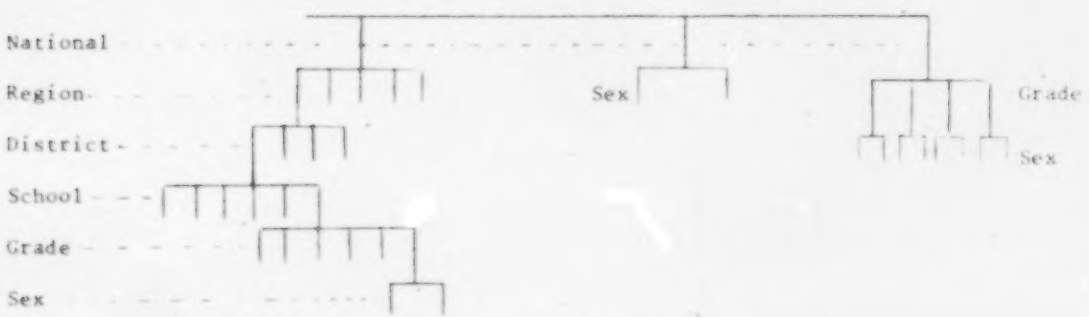
Ineffectiveness in an educational system is represented by failure to ensure that each pupil reaches the target achievement set for him. If the goal of a system is to secure the ultimate passage of all pupils enrolled in a cycle through all grades in that cycle, then ineffectiveness is represented by drop-out in the course of the cycle. In such a system the same target level of achievement is set for all pupils and it might be argued that no system yet has the facilities to ensure that this target is reached, so that functionally it sets different achievement levels, and drop-out following failure in a grade is in practice the recognition that the system gives to lower that final grade standards. Drop-out which occurs in the course of a year more probably represents ineffectiveness of the school system in that its holding-power is less that perfect. Ineffectiveness would then be measured by the difference in grade levels attained by drop-outs from those which they would otherwise have achieved.

Educational systems will obviously differ in the variance and mean level of aptitude of their pupils - higher variance and lower means in systems having universal education and being non-selective than in those which are selective and which educate only a proportion of the population of school age. Other factors like the readiness of children to begin formal education will also produce differences between systems. Consequent upon these differences as well as upon the quality of learning opportunity in school, there will be differences in the mean level and variance of achievement amongst 'fixed duration' systems and in the mean and the variance of number of years spent in school in 'fixed level' systems.

In measuring the efficiency of education, methods which ignore the mean and variance of aptitude will tend to inflate estimates of wastage. Granted that aptitude is difficult to measure, attempts should be made to arrive at approximations. In 'fixed level' systems, one possible device would be to use the performance of graduates from the cycle, in their first year to obtain a mean and variance of grade 1 passing and failing which might be regarded as a measure of aptitude for beginning formal education.

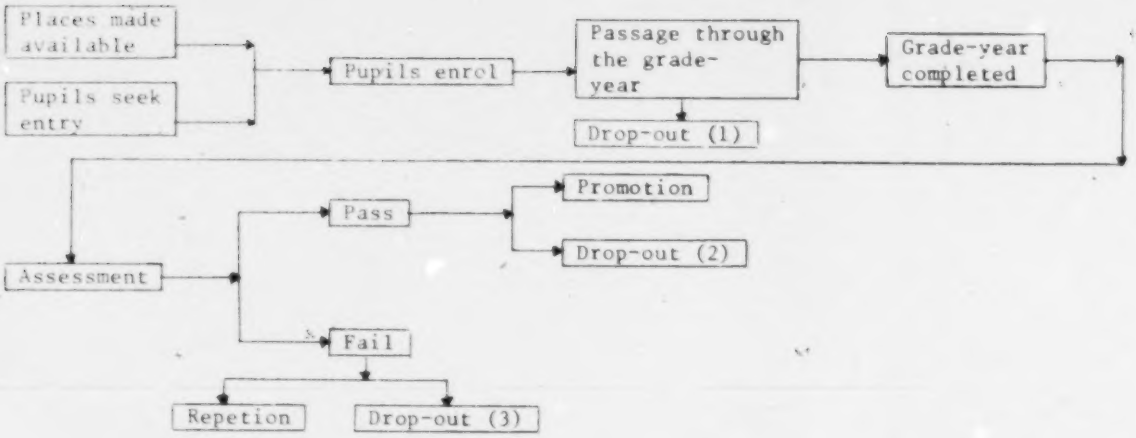
The production of estimates of wastage and inefficiency must relate to the educational action which ought to follow through the medium of policy decision. To know that wastage is high at a particular level is not enough, unless it also locates the specific sector of the system where it occurs in such a way that action can be taken. Breakdowns of wastage estimates into urban/rural, boys/girls, contribution is not sufficient, since school systems are administered directly in much smaller units. Moreover, the highly specific factors of community practices, buildings, equipment, teachers - are masked by large sector study and are only discernible in a highly localised context. In the last analysis, each school must be separately identifiable and each grade within that school. It then becomes possible to direct actions specifically. A network of estimates resembling the following is required in which the levels correspond to the levels of administrative control, if planning is to be effective.

LEVELS AT WHICH INDICES MIGHT BE DERIVED



The aim should be to produce comparable estimates at each level of the system in a form which relates directly to decision making. The conventional use of pupil flow events to estimate wastage does not reflect educational decision, precisely because it involves non-educational decision. If we draw a flow chart of action this becomes evident.

ACTION FLOW



There are three conditions of drop out and one condition of repetition. The three types of drop out will be identified in the rest of the paper as types one, two and three as they are shown in the flow chart. The first two types may be regarded as the result of extra-system factors, reflecting the holding power of the school but not its internal efficiency. It should be known, but distinguished from type three. Type three drop out may be regarded as a direct consequence of the decision that the pupil has failed. In so far as the decision as to whether the pupil has passed or failed also determines repetition and promotion, it may be regarded as the focal determining factor in relevant wastage. In a totally different sense, it also represents wastage, since it reflects a judgement that the last year spent in school has been unprofitable. The educational decision as to whether a pupil has satisfied the grade requirement is not only focal in the flow of action, but it is also an event over which policy decisions can exercise control. If estimates of efficiency and wastage related directly to graduation/non-graduation from grades, not only would estimates be more accurate, but more relevant policy decisions could follow. It would allow, for example, a distinction between the dependence of enrolments in a grade upon successful study in the previous grade or upon availability of places or upon extra system factors.

The next section of this paper is concerned with an attempt to derive estimates of wastage and efficiency from the observation of graduation/non-graduation events in a system.

The Proposed Model

Assumptions

The following data are available for a cohort for any given cycle in each year of the duration of that cohort.

1. The number of graduates from each grade.
2. The number of pupils completing each grade who failed that grade.
3. The number of pupils who enrolled in each grade by the grade from which they came.

Definitions

O_{ij} is the number of graduates from grade i in year j.

$\emptyset_{ij}$ is the number of pupils completing grade i in year j, who failed to graduate from that grade.

OE_{ij} is the number of pupils enrolled in grade i in year j, who graduated from grade i -1 in year j -1.

Let OE_{11} be the enrolment in grade 1 in the first year of the cohort.

$\emptyset E_{ij}$ is the number of pupils enrolled in grade i in year j who failed that grade in vear j -1.

g is the number of grades in the cvlce.

l is the final grade of the cycle.

a is the last year of the cohort's duration in the cycle.

Inferences

(i) $\sum_{i=1}^{l}\sum_{j=1}^{a}\left[(OE_{ij}+\emptyset E_{ij})-(O_{ij}+\emptyset_{ij})\right]$

is the total drop out of type 1 and is attributable to extra-system factors.

(ii) $\sum_{i=1}^{l}\sum_{j=1}^{a}(O_{ij}) - \sum_{i=2}^{l}\sum_{j=2}^{a} OE_{ij}$

is the total drop out of type 2 and is also attributed to extra-system factors.

Thus wastage attributable to the system, is computed after drop out of types 1 and 2 have been excluded.

(iii) $\sum_{i=1}^{l}\sum_{j=1}^{a}(O_{ij}+\emptyset_{ij})$ is the input to the system.

(iv) $\sum_{j=1}^{a} O_{lj}$ is the output from the system.

(v) $\frac{(iii)}{(iv)}$ is the input/output relation.

(vi) $\frac{(v)}{g}$ is the efficiency ratio. This is so because the optimum efficiency relation is reached when (v) equals g.

Both drop out and repetition are regarded as consequences of wastage by this model. Nevertheless interest may lie in apportioning wastage to its consequences and the following inferences are drawn.

(vii) $\sum_{i=1}^{l}\sum_{j=1}^{a} \emptyset E_{ij}$ is wastage associated with consequent repetition.

(viii) $\sum_{i=1}^{l}\sum_{j=1}^{a} \emptyset_{i-1,j-1} - \emptyset E_{ij}$ is wastage ascribable to consequent, relevant drop out. Likewise,

(ix) $\left(\sum_{i=1}^{l}\sum_{j=1}^{a} \emptyset_{ij}\right)$ - (viii) is wastage ascribable to graduates from the cycle.

The model appears neat enough and accounts for the events in which the main interest lies. We can summarize its virtues as follows :

1. It identifies and quantifies total wastage from a cycle and apportions wastage to drop outs, repeaters and graduates.

2. It enables drop out not attributable to within-system inefficiency alone, to be identified and quantified.

3. It expresses wastage in terms of failure to reach the educational standards of each grade, although involving completion of that grade.

4. It enables an efficiency index for the total system or any of its parts to be prepared.

The next and crucial issue arises in arriving at estimates of the parameters of the model. Clearly, the model minimally supposes that some form of cohort coding be employed. Unfortunately, its full benefit can only be derived by introducing two data collection points in each year. The system adopted would be similar to the following :

1. Each pupil is coded on enrolment in a grade by the grade from which he came and the beginning year of the cohort to which he belongs, viz : E_{iy}

2. Each pupil is coded on completion of a grade by the last grade he passed (his present grade if he were successful in it), and by the beginning year of the cohort, viz : P_{iy}

All values of OE_{ij} and $\emptyset E_{ij}$ can be determined from E_{iy}, and P_{iy} leads to all values of O_{ij} and $\emptyset_{ij}$.

A poor substitute for this would be to use the enrolment coding E_{iy} only and to have only one data collection point. Although total drop out could be determined from it, wastage specifically relevant to wastage could not. Moreover, graduation/non-graduation from the last grade of the cycle could not be determined. It would seem preferable to introduce the more elaborate pupil accounting system.

The suggested model raises interesting questions concerning the assessment of wastage in countries that operate, either legalistically or de facto, automatic promotion. In many of these countries, achievement tests are available which permit assessment of a child according to his grade or his age. Scores are expressed as deviations from a mean of normally distributed measures and usually the regression of scores upon age is used to provide adjusted scores enabling each child to be compared with others of the same age.

Let us take an argument which assumes the administration of a single criterion test to a cohort in the final year of a cycle. All pupils have taken a standardized achievement test and upon the raw score distribution, a criterion level of score has been imposed which represents satisfactory achievement of the grade. Attaining this score level would then be equivalent to graduations from the cycle, and could be treated in the same way. However, the imposition of a single criterion point on the scale, ignores the range of individual difference present in an age group and represents a very crude target for achievement unless it also specifies the proportion of the age group expected to reach it. A more realistic and operational approach would be to impose a number of raw score criterion levels upon the distribution and to relate these to target quantiles of the sample. Thus the criterion raw scores might be related to target deciles. The proportion of the sample which would be regarded as graduating would then be determined by the proportion at least satisfying the target deciles.

Fig. 1 illustrates.

Cumulative Target Deciles	90	80	70	60	50	40	30	20	10
Raw score scale									
Obtained cumulative proportions	.85	.75	.70	.50	.35	.30	.20	.15	.05
Difference	+ 5	+ 5	0	+10	+15	+10	+10	+ 5	+ 5 --- Z

The maximum deficit of the obtained cumulative proportions in relation to the target cumulative proportions represents the proportion failing to reach the target level, while the complement is the proportion graduating. One way of costing the 'failure' rate would be to regard the unit cost as one year for each pupil. In which case over the four years of a cycle, the proportion graduating in the final year is the output and the input is 1.00. The efficiency index is then $\frac{1}{p}$.

It is however most likely that multiple criteria would be set in which there might be as many as five test results for each child. Rather than deal with the complexities of a profile analysis, it would be preferable to combine the criteria into a composite by

applying pre-determined weights. It would also be possible to produce an expression of wastage not merely in terms of failure to reach targets, but in terms of a time based cost. Each proportionate failure to reach the target level can be costed in terms of the time expected to be taken to recover, since the regression of score upon month of age is known and cross sectional age-score regression is a useful approximation to the regression of score upon age calculated in longitudinal follow up.

In practice this method lends itself most easily to a sample survey of the target population, which if stratified random sampling techniques are employed and within school pupil sampling is permitted can yield tolerable limits of sampling error and location of those parts of the system most subject to wastage and inefficiency.

The example referred to estimates based upon the behaviour of the cohort in the final year. While this would be sufficient in a system which virtually excludes repetition and drop out, clearly it has disadvantages in a system which allows either. It would indeed be preferable to construct a continuous achievement scale covering all grades of a cycle such that a child's performance could be estimated on it irrespective of the grade in which he was. Such a suggestion raises critical problems of curriculum evaluation and assumes that by some means an overall scale could be constructed. While technically there are few problems there would undoubtedly be objections raised by educators to the compression of diverse aspects of the curriculum into a single criterion measure. However, so long as the device were clearly employed to monitor the behaviour of the system and to assist planning operations it need have no undesirable consequences. It would be preferable to abandon the conventional methods of test standardisation and construction which rely upon deviations from a sample mean and to produce tests which were criterion referenced (see Glazer 1967). Methods of absolute scaling which are adaptable to criterion referenced assessment have been worked out by Rasch (1960) and others.

Even in those systems where currently no tests are available and which admit of repetition and drop out, it would be possible to carry out modes of grade assessment which permitted individual variation in possible achievement to be indicated. There is no denying the difficulties that would be encountered in introducing the cohort coding system let alone the introduction of differential levels of grade achievement. Nevertheless the mood of nations is better oriented towards advance in the reporting and control of the behaviour of educational system than it has ever been before. Perhaps an experimental introduction of the methods described here might be worth while.

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Appendix one

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